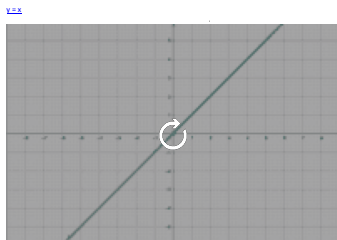


7.1 $D = \{(x, y) \in \mathbb{R}^2 : 0 \leq y \leq x, 0 \leq x \leq 1\}$ $\begin{matrix} u = 0 \\ x - y = 0 \end{matrix}$

a) $\iint_D (x+y) d(x,y)$ $y=x \xrightarrow{x=1}$

$$\begin{aligned} & \int_0^1 \int_y^1 (x+y) dx dy \\ &= \int_0^1 \left[\frac{x^2}{2} + yx \right]_y^1 dy \\ &= \int_0^1 \left(\frac{1}{2} + y^2 - \left(\frac{1}{2} + y \right) \right) dy \\ &= \int_0^1 \left(\frac{3y^2}{2} - \frac{(1+y)}{2} \right) dy \end{aligned}$$



$$= \frac{1}{2} \int_0^1 (3y^2 - 2y - 1) dy = \frac{1}{2} \left(\frac{3y^3}{3} - \frac{2y^2}{2} - y \right) \Big|_0^1 = \frac{1}{2} (1 - 1 - 1) = -\frac{1}{2}$$

b) $x = u+v$; $y = u-v$

$$\begin{cases} x = u+v \\ y = u-v \end{cases} \Leftrightarrow \begin{cases} u = x-v \\ v = u-y \end{cases} \Leftrightarrow \begin{cases} u = x - (u-y) \\ v = u-y \end{cases} \Leftrightarrow \begin{cases} 2u = x+y \\ v = \frac{x+y}{2} - y \end{cases}$$

* $\begin{cases} u = \frac{x+y}{2} \\ v = \frac{x-y}{2} \end{cases}$

$\Phi: D^* \rightarrow D$

$(u,v) \rightarrow \left(\frac{x+y}{2}, \frac{x-y}{2} \right)$

$(x,y) \downarrow$
 $P(0,0) \rightarrow \begin{cases} x=0 \\ y=0 \end{cases} \rightarrow \begin{cases} u=0 \\ v=0 \end{cases}$

$P(1,1) \rightarrow \begin{cases} x=1 \\ y=1 \end{cases} \rightarrow \begin{cases} u=1 \\ v=0 \end{cases}$

$P(1,0) \rightarrow \begin{cases} x=1 \\ y=0 \end{cases} \rightarrow \begin{cases} u=\frac{1}{2} \\ v=\frac{1}{2} \end{cases}$

$v=u$



$\frac{\partial u}{\partial x} = \frac{1}{2}$; $\frac{\partial u}{\partial y} = \frac{1}{2}$; $\frac{\partial v}{\partial x} = \frac{1}{2}$; $\frac{\partial v}{\partial y} = -\frac{1}{2}$

$J(u,v) = \left| \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} \right| = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$

$\Rightarrow \iint_D (x+y) d(x,y) = \iint_{D^*} u+v + u-v \left(\frac{1}{2} \right) d(u,v)$

$= \iint_{D^*} u d(u,v)$

$= \int_0^1 \int_v^{1-v} u du dv$

$= \int_0^1 \left[\frac{u^2}{2} \right]_v^{1-v} dv = \frac{1}{2} \int_0^1 (1-v)^2 - v^2 dv = \frac{1}{2} \int_0^1 1 - 2v dv = \frac{1}{2} (v - v^2) \Big|_0^1 = \frac{1}{2} \left(\frac{1}{2} - \frac{1}{4} \right) = \frac{1}{4} \left(\frac{1}{2} \right) = \frac{1}{8}$

10)

a) $\iint_D (x^2 + y^2) d(x,y)$ $D = \{(x,y) \in \mathbb{R}^2 : -1 \leq x-y \leq 0, -1 \leq x+y \leq 0\}$

$x-y = -1 \Leftrightarrow y = x+1$; $x+y = -1 \Leftrightarrow y = -x-1$

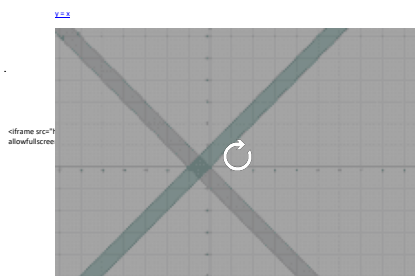
$x-y = 0 \Leftrightarrow y = x$ $x+y = 0 \Leftrightarrow y = -x$

$u = x-y \rightarrow -1 \leq u \leq 0$

$v = x+y \rightarrow -1 \leq v \leq 0$

$\Phi: D^* \rightarrow D$
 $(u,v) \rightarrow (x-y, x+y)$

$(u,v) = \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$



1 1 1 1 1

$J_D(u, v)$

$$\begin{cases} u = x - y \\ v = x + y \end{cases} \Leftrightarrow \begin{cases} u = x - (v - x) \\ y = v - x \end{cases} \Leftrightarrow \begin{cases} u = 2x - v \\ y = v - x \end{cases}$$

$$\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$$

$$\Rightarrow |J_D(x, y)| = \frac{1}{2}$$

$$\Leftrightarrow \begin{cases} x = \frac{u+v}{2} \\ y = v - \frac{u+v}{2} \end{cases} \Leftrightarrow \begin{cases} x = \frac{u+v}{2} \\ y = \frac{v-u}{2} \end{cases}$$

$$\iint_D (x^2 + y^2) d(x, y) = \iint_{D^*} \left(\frac{u+v}{2}\right)^2 + \left(\frac{v-u}{2}\right)^2 \cdot 2 d(u, v) = \frac{1}{2} \iint_{-1}^1 ((u+v)^2 + (v-u)^2) du dv \Rightarrow$$

$$\Rightarrow \frac{1}{2} \int_{-1}^1 \int_{-1}^1 (u^2 + 2uv + v^2 + v^2 - 2uv + u^2) du dv = \frac{1}{2} \int_{-1}^1 \int_{-1}^1 (2u^2 + 2v^2) du dv = \int_{-1}^1 \left[\frac{2}{3} u^3 + v^2 u \right]_{-1}^1 dv \Rightarrow$$

$$\Rightarrow \int_{-1}^1 \left[-\frac{1}{3} + v^2 \right] dv = \left[-\frac{1}{3} v + \frac{v^3}{3} \right]_{-1}^1 = \left[\frac{v^3 - v}{3} \right]_{-1}^1 = \left[-\frac{1+1}{3} \right] = -\frac{2}{3} \checkmark$$

Exercício 7.10 Calcule os seguintes integrais, usando uma mudança de variáveis adequada:

a) $\iint_D (x^2 + y^2) d(x, y)$, onde $D = \{(x, y) \in \mathbb{R}^2 : -1 \leq x - y \leq 0, -1 \leq x + y \leq 0\}$;

b) $\iint_D (x - y) d(x, y)$, onde $D = \{(x, y) \in \mathbb{R}^2 : -x \leq y \leq 3 - x, 2x - 2 \leq y \leq 2x\}$.

b) $\iint_D (x - y) d(x, y)$, $D = \{(x, y) \in \mathbb{R}^2 : -x \leq y \leq 3, 2x - 2 \leq y \leq 2x\}$

$u = x + y \rightarrow \text{linear}$

$v = 2x - y$

$(u, v) = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -1 - 2 = -3$

$\Rightarrow J_D(x, y) = -\frac{1}{3}$

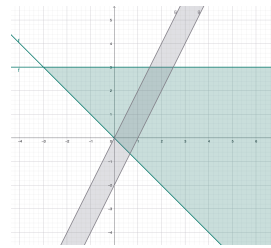
$\det \left(\frac{\partial(x, y)}{\partial(u, v)} \right) = \frac{1}{\det \left(\frac{\partial(u, v)}{\partial(x, y)} \right)}$

$$\begin{cases} u = x + y \\ v = 2x - y \end{cases} \Leftrightarrow \begin{cases} y = u - x \\ v = 2x - u + x \end{cases} \Leftrightarrow \begin{cases} y = u - x \\ v = 3x - u \end{cases}$$

$$\rightarrow \begin{cases} y = u - \frac{u+v}{3} \\ x = \frac{u+v}{3} \end{cases} \Leftrightarrow \begin{cases} y = \frac{2u-v}{3} \\ x = \frac{u+v}{3} \end{cases}$$

$$\int_0^3 \int_0^2 \left(\frac{u+v}{3} - \frac{2u-v}{3} \right) \left(\frac{1}{3} \right) dv du = \frac{1}{9} \int_0^3 \int_0^2 (2v - u) dv du$$

$$= \frac{1}{9} \int_0^3 \left[\frac{2v^2}{2} - uv \right]_0^2 du = \frac{1}{9} \int_0^3 (4 - 2u) du = \frac{1}{9} \left[4u - \frac{2u^2}{2} \right]_0^3 = \frac{1}{9} (12 - 9) = 4 - 3 = 1$$



Exercício 7.11 Calcule os seguintes integrais, usando coordenadas polares.

a) $\int_0^{2R} \int_0^{\sqrt{2Rx-x^2}} (x^2 + y^2) dy dx$;

b) $\int_0^1 \int_{x^2}^x dy dx$.

7.11 a) $\int_0^{2R} \int_0^{\sqrt{2Rx-x^2}} (x^2 + y^2) dy dx$

$D = D^* \rightarrow D$

$(r, \theta) \rightarrow (R \cos \theta, R \sin \theta)$

$x = R \cos \theta$

$y = R \sin \theta$

$x^2 + y^2 = R^2$

$\Rightarrow R^2 \cos^2 \theta + R^2 \sin^2 \theta = 2Rx$

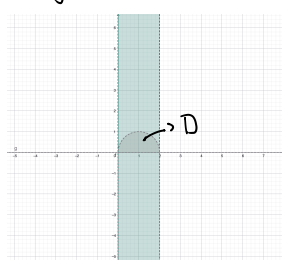
$\Rightarrow R^2 (\cos^2 \theta + \sin^2 \theta) = 2Rx$

$\Rightarrow R^2 = 2Rx \cos \theta$

$\Rightarrow R(1 - 2R \cos \theta) = 0$

$\Rightarrow R = 0 \vee R = 2R \cos \theta$

$0 < x < 2R$
 $0 < y < \sqrt{2Rx - x^2}$



$y = \sqrt{2Rx - x^2}$

$y^2 = 2Rx - x^2$

$\Leftrightarrow x^2 + y^2 = 2Rx$

$\Leftrightarrow x^2 - 2Rx + y^2 = 0$

$\Leftrightarrow (x - R)^2 - R^2 + y^2 = 0$

$\Leftrightarrow (x - R)^2 + y^2 = R^2$

$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta = \cos^2 \theta - (1 - \cos^2 \theta) = 2\cos^2 \theta - 1 \rightarrow \cos^2 \theta = \frac{\cos(2\theta) + 1}{2}$

$$\int_0^{\frac{\pi}{2}} \int_0^{R \cos \theta} (r^2 \cos^2 \theta + r^2 \sin^2 \theta) r dr d\theta = \int_0^{\frac{\pi}{2}} \int_0^{R \cos \theta} r^3 dr d\theta = \int_0^{\frac{\pi}{2}} \frac{r^4}{4} \Big|_{r=0}^{R \cos \theta} d\theta \rightarrow$$

$$\rightarrow \frac{1}{4} \int_0^{\frac{\pi}{2}} 16R^4 \cos^4 \theta d\theta = 4R^4 \int_0^{\frac{\pi}{2}} \left(\frac{\cos(2\theta) + 1}{2} \right)^2 d\theta = R^4 \int_0^{\frac{\pi}{2}} (\cos(2\theta) + 1)^2 d\theta$$

$$\rightarrow R^4 \int_0^{\frac{\pi}{2}} \cos^2(2\theta) + 2\cos(2\theta) + 1 d\theta = R^4 \left(\int_0^{\frac{\pi}{2}} \frac{\cos(4\theta) + 1}{2} d\theta + \int_0^{\frac{\pi}{2}} 2\cos(2\theta) d\theta + \int_0^{\frac{\pi}{2}} 1 d\theta \right)$$

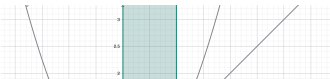
$$\rightarrow R^4 \left(\frac{\pi}{2} + \left[\frac{\sin(4\theta)}{4} \right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{1 + \cos(2\theta)}{2} d\theta \right) = R^4 \left(\frac{\pi}{2} + \frac{1}{2} \left(\int_0^{\frac{\pi}{2}} d\theta + \frac{1}{4} \int_0^{\frac{\pi}{2}} 4\cos(4\theta) d\theta \right) \right)$$

$$\rightarrow R^4 \left(\frac{\pi}{2} + \frac{\pi}{4} + \frac{1}{4} [\sin(4\theta)]_0^{\frac{\pi}{2}} \right) = \frac{3\pi R^4}{4}$$

b) $\int_0^1 \int_{x^2}^x dy dx$

$x^2 \leq y \leq x$

$0 \leq x \leq 1$



$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$y = x \Rightarrow r \sin \theta = r \cos \theta$$

$$\Rightarrow \tan \theta = 1$$

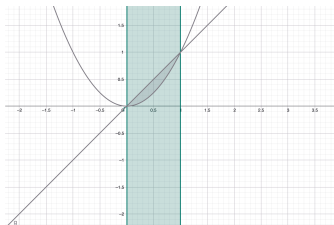
$$\Rightarrow \theta = \frac{\pi}{4}$$

$$y = x^2 \Rightarrow r \sin \theta = r^2 \cos^2 \theta$$

$$\Rightarrow \frac{\sin \theta}{\cos^2 \theta} = r$$

$$\int_0^{\frac{\pi}{4}} \int_0^{\frac{\sin \theta}{\cos^2 \theta}} r \, dr \, d\theta = \int_0^{\frac{\pi}{4}} \frac{r^2}{2} \Big|_0^{\frac{\sin \theta}{\cos^2 \theta}} d\theta = \frac{1}{2} \int_0^{\frac{\pi}{4}} \frac{\sin^2 \theta}{\cos^4 \theta} d\theta = \frac{1}{2} \int_0^{\frac{\pi}{4}} \tan^2 \theta \frac{1}{\cos^2 \theta} d\theta$$

$$\rightarrow \frac{1}{2} \int_0^{\frac{\pi}{4}} \tan^2 \theta \sec^2 \theta d\theta = \frac{1}{2} \int_0^1 u^2 du = \frac{1}{2} \left[\frac{u^3}{3} \right]_0^1 = \frac{1}{6}$$



$$\frac{1}{\cos^2 \theta} = \sec^2 \theta$$

$$u = \tan \theta$$

$$du = \sec^2 \theta d\theta$$

$$\theta = 0 \rightarrow u = 0$$

$$\theta = \frac{\pi}{4} \rightarrow u = 1$$

Exercício 7.12 Calcule

$$\iint_D \sqrt{x^2 + y^2} \, d(x, y),$$

$$\text{sendo } D = \{(x, y) \in \mathbb{R}^2 : 4 \leq x^2 + y^2 \leq 9\}.$$

7.12

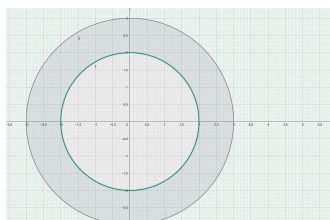
$$D = \{(x, y) \in \mathbb{R}^2 : 4 \leq x^2 + y^2 \leq 9\}$$

$$\iint_D \sqrt{x^2 + y^2} \, d(x, y)$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$\sqrt{x^2 + y^2} = \sqrt{r^2 \cos^2 \theta + r^2 \sin^2 \theta} = \sqrt{r^2} = r$$



$$\int_2^3 \int_0^{2\pi} r^2 \, d\theta \, dr = \int_2^3 r^2 \, dr \int_0^{2\pi} d\theta = \left[\frac{r^3}{3} \right]_2^3 \times 2\pi = \left(\frac{27}{3} - \frac{8}{3} \right) \times 2\pi = \left(9 - \frac{8}{3} \right) 2\pi = \frac{34}{3} \pi$$

Exercício 7.13 Calcule

$$\iint_D xy^3 \, d(x, y),$$

$$\text{sendo } D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 \text{ e } x \geq 0 \text{ e } y \geq 0\}.$$

$$\iint_D xy^3 \, d(x, y) \quad D = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1, x \geq 0, y \geq 0\}$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$xy^3 \Rightarrow r^4 \cos \theta \sin^3 \theta$$

$$\int_0^{\frac{\pi}{2}} \int_0^1 r^5 \cos \theta \sin^3 \theta \, dr \, d\theta = \int_0^{\frac{\pi}{2}} \cos \theta \sin^3 \theta \, d\theta \int_0^1 r^5 \, dr$$

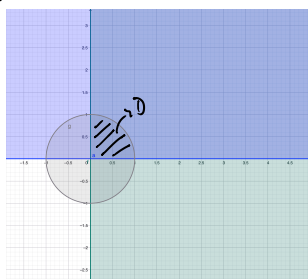
$$\rightarrow \left(\int_0^1 u^3 \, du \right) \left(\frac{r^6}{6} \Big|_0^1 \right) = \frac{u^4}{4} \Big|_0^1 \times \frac{r^6}{6} \Big|_0^1 = \frac{1}{4} \times \frac{1}{6} = \frac{1}{24}$$

$$u = \sin \theta$$

$$du = \cos \theta d\theta$$

$$\theta = 0 \rightarrow u = 0$$

$$\theta = \frac{\pi}{2} \rightarrow u = 1$$



Exercício 7.14 Calcule, usando coordenadas polares, o volume dos sólidos limitados por:

a) $z = 4 - x^2 - y^2$ e $z = 0$;

b) $z = 6 - x^2 - y^2$ e $z = \sqrt{x^2 + y^2}$.

7.14

$$1) \quad z = 4 - x^2 - y^2, \quad z = 0$$

$$4 - x^2 - y^2 = 0 \Rightarrow x^2 + y^2 = 4$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$z = 4 - x^2 - y^2$$

$$z = 4 - (x^2 + y^2)$$

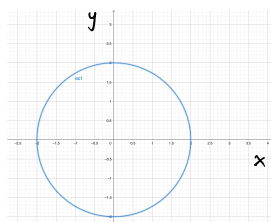
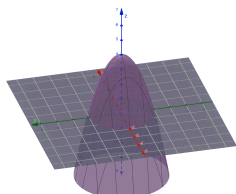
$$z = 4 - r^2$$

$$4 - r^2 = 0 \Rightarrow r^2 = 4$$

$$\Rightarrow r = \pm 2$$

$$r \geq 0 \rightarrow r = 2$$

$$\int_0^{2\pi} \int_0^2 (4 - r^2) \, dr \, d\theta$$



$$\cdot 2\pi \left(4r - \frac{r^3}{3} \right) \Big|_0^2$$

$$\cdot 2\pi \left(8 - \frac{8}{3} \right) = 2\pi \left(\frac{16}{3} \right) = \frac{32}{3} \pi$$

$$\cdot z = 6 - x^2 - y^2, \quad z = \sqrt{x^2 + y^2}$$

$$\cdot -x^2 - y^2 = \sqrt{x^2 + y^2} \quad *$$

$$\cdot r \cos \theta$$

$$\cdot r \sin \theta$$

$$\rightarrow 6 - r^2 = r \quad (= r^2 + r - 6 = 0)$$

$$\cdot \left(r + \frac{1}{2} \right)^2 - \frac{1}{4} - 6 = 0$$

$$\cdot \left(r + \frac{1}{2} \right)^2 = \frac{25}{4}$$

$$\cdot r + \frac{1}{2} = \pm \frac{5}{2} \quad (= r = -3 \vee r = 2)$$

$$r \geq 0 \rightarrow r = 2$$

$$\cdot \int_0^{2\pi} \int_0^2 (6 - x^2 - y^2 - \sqrt{x^2 + y^2}) dx dy = \int_0^{2\pi} \int_0^2 (6 - r^2 - r) r dr d\theta$$

$$\rightarrow \int_0^{2\pi} \left(\frac{6r^2}{2} - \frac{r^4}{4} - \frac{r^3}{3} \right) \Big|_0^2 d\theta = \int_0^{2\pi} \left(3r^2 - \frac{r^4}{4} - \frac{r^3}{3} \right) \Big|_0^2 d\theta$$

$$\rightarrow 2\pi \left(12 - 4 - \frac{8}{3} \right) = 2\pi \left(\frac{36}{3} - \frac{12}{3} - \frac{8}{3} \right) = 2\pi \left(\frac{16}{3} \right) = \frac{32}{3} \pi$$

exercício 7.1 Seja R o retângulo $[0, 1] \times [1, 2]$. Calcule os seguintes integrais:

a) $\iint_R (x^3 + y^2) dx dy$;

c) $\iint_R (xy)^2 \cos x^3 dx dy$;

b) $\iint_R ye^{xy} dx dy$;

d) $\iint_R \ln((x+1)y) dx dy$.

1.1 $R = [0, 1] \times [1, 2]$

$$\cdot \iint_R (x^3 + y^2) dx dy = \int_1^2 \int_0^1 (x^3 + y^2) dx dy$$

$$\rightarrow \int_1^2 \left[\frac{x^4}{4} + xy^2 \right]_0^1 dy = \int_1^2 \left(\frac{1}{4} + y^2 \right) dy$$

$$\rightarrow \left[\frac{y}{4} + \frac{y^3}{3} \right]_1^2 = \frac{1}{2} + \frac{8}{3} - \frac{1}{4} - \frac{1}{3} = \frac{6 + 32 - 3 - 4}{12} = \frac{31}{12} \checkmark$$

$$\cdot \iint_R ye^{xy} dx dy = \int_1^2 \int_0^1 ye^{xy} dx dy = \int_1^2 e^{xy} \Big|_0^1 dy = \int_1^2 (e^y - 1) dy$$

$$\rightarrow e^y - y \Big|_1^2 = e^2 - 2 - e + 1 = e^2 - e - 1 \checkmark$$

exercício 7.2 Calcule os seguintes integrais, esboçando as regiões de integração:

a) $\int_0^1 \int_0^x dy dx$;

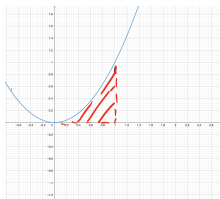
c) $\int_0^1 \int_1^2 (x+y) dx dy$;

b) $\int_1^2 \int_{x^2}^{2x+1} dy dx$;

d) $\int_0^1 \int_{x^2}^1 y dy dx$.

$$\cdot \int_0^1 \int_0^x dy dx$$

$$\cdot x^2 dx = \frac{x^3}{3} \Big|_0^1 = \frac{1}{3}$$



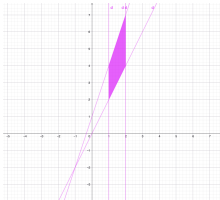
$$\cdot \int_1^2 \int_{2x}^{3x+1} dy dx$$

$$\cdot 3x+1-2x dx$$

$$\cdot \int_1^2 x+1 dx = \frac{x^2}{2} + x \Big|_1^2$$

$$2+2 - \frac{1}{2} - 1 = 3 - \frac{1}{2} =$$

$$\frac{5}{2}$$



$$D = \{(x, y) \in \mathbb{R}^2 : 1 \leq x \leq 2, 2x \leq y \leq 3x+1\}$$

Exo 7.4 Esboce a região de integração e inverta a ordem de integração em cada um dos seguintes integrais:

a) $\int_{-2}^0 \int_0^{1-x^2} f(x, y) dy dx$;

b) $\int_0^1 \int_0^{1-x^2} f(x, y) dy dx$;

c) $\int_0^1 \int_{x^2}^{2x+1} f(x, y) dy dx$;

d) $\int_0^1 \int_{x^2}^{1-x^2} f(x, y) dy dx$;

e) $\int_{-2}^0 \int_{1-x^2}^{2x+1} f(x, y) dy dx$;

f) $\int_0^1 \int_{2x}^{3x+1} f(x, y) dy dx$;

g) $\int_0^1 \int_{x^2}^{1-x^2} f(x, y) dy dx$;

h) $\int_0^1 \int_{x^2}^{1-x^2} f(x, y) dy dx$;

i) $\int_0^1 \int_{x^2}^{1-x^2} f(x, y) dy dx + \int_0^1 \int_{x^2}^{1-x^2} f(x, y) dy dx$;

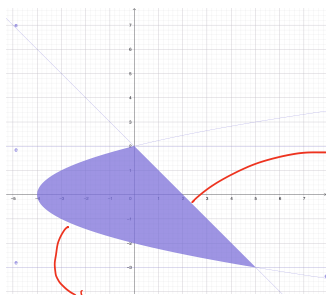
j) $\int_{-2}^0 \int_{1-x^2}^{2x+1} f(x, y) dy dx + \int_0^1 \int_{x^2}^{1-x^2} f(x, y) dy dx$;

$$1) \int_{-3}^2 \int_{-4+y^2}^{4-y} f(x,y) dx dy$$

$$2) \int_{-4}^0 \int_{-4+y^2}^{4-y} f(x,y) dy dx +$$

$$3) \int_0^2 \int_{-4+y^2}^{4-y} f(x,y) dy dx$$

$$D = \{(x,y) \in \mathbb{R}^2 : -4+y^2 \leq x \leq 4-y, -3 \leq y \leq 2\}$$



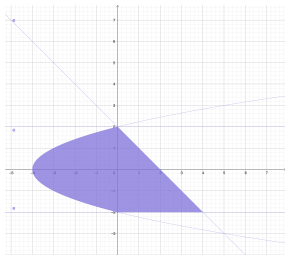
$$x = y^2 - 4 \Leftrightarrow y = \pm \sqrt{x+4}$$

$$1) \int_{-2}^2 \int_{y^2-4}^{2-y} f(x,y) dx dy$$

$$2) \int_{-4}^0 \int_{-4+y^2}^{4-y} f(x,y) dy dx +$$

$$3) \int_0^2 \int_{-4+y^2}^{4-y} f(x,y) dy dx$$

$$D = \{(x,y) \in \mathbb{R}^2 : y^2 - 4 \leq x \leq 2 - y, -2 \leq y \leq 2\}$$



Exercício 7.5 Representa graficamente o conjunto D e calcule, recorrendo a integrais duplos, a sua área:

- a) $D = \{(x,y) \in \mathbb{R}^2 : 1 \leq x \leq 2, x \leq y \leq x^2\}$;
b) $D = \{(x,y) \in \mathbb{R}^2 : -y^2 \leq x \leq y^2, 0 \leq y \leq 1\}$.

5

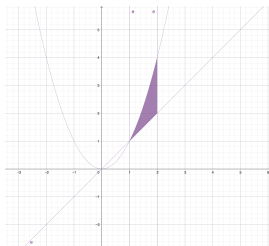
$$1) D = \{(x,y) \in \mathbb{R}^2 : 1 \leq x \leq 2, x \leq y \leq x^2\}$$

$$\text{Área}(D) = \int_1^2 (x^2 - x) dx$$

$$= \iint_D d(x,y)$$

$$\int d(x,y) = \int_1^2 \int_x^{x^2} dy dx = \int_1^2 (x^2 - x) dx$$

$$\left. \frac{x^3}{3} - \frac{x^2}{2} \right|_1^2 = \frac{8}{3} - 2 - \frac{1}{3} + \frac{1}{2} = \frac{7}{3} - \frac{3}{2} = \frac{5}{6}$$



Exercício 7.6 Determine a área limitada pelas curvas definidas por $x^2 + y^2 = 2x$, $x^2 + y^2 = 4x$, $y = x$ e $y = 0$.

$$x^2 + y^2 = 2x \Leftrightarrow x^2 - 2x + y^2 = 0$$

$$\Leftrightarrow (x-1)^2 + y^2 = 1$$

$$x^2 + y^2 = 4x \Leftrightarrow x^2 - 4x + y^2 = 0$$

$$\Leftrightarrow (x-2)^2 + y^2 = 4$$

$$\text{Área}(D) = \iint_D d(x,y)$$

$$x^2 + y^2 = r^2 \Rightarrow r^2 = 2(r \cos \theta)$$

$$\Rightarrow r = 2 \cos \theta$$

$$x^2 + y^2 = 4x$$

$$r^2 = 4(r \cos \theta) \Rightarrow r = 4 \cos \theta$$

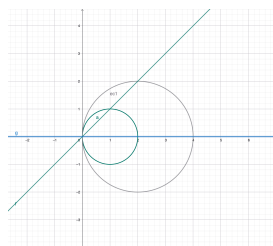
$$\Rightarrow r = 4 \cos \theta$$

$$\int_0^{\pi/4} \int_{2 \cos \theta}^{4 \cos \theta} r dr d\theta = \int_0^{\pi/4} \left. \frac{r^2}{2} \right|_{2 \cos \theta}^{4 \cos \theta} d\theta$$

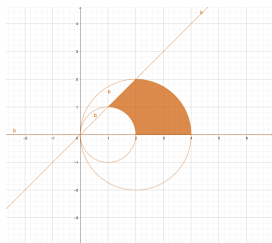
$$\int_0^{\pi/4} \frac{16 \cos^2 \theta}{2} - \frac{4 \cos^2 \theta}{2} d\theta = \int_0^{\pi/4} 6 \cos^2 \theta d\theta$$

$$6 \int_0^{\pi/4} \frac{\cos(2\theta) + 1}{2} d\theta = 3 \left(\frac{1}{2} \int_0^{\pi/4} 2 \cos(2\theta) d\theta + \frac{\pi}{4} \right)$$

$$3 \left[\frac{1}{2} \left(\sin \frac{\pi}{2} \right) + \frac{\pi}{4} \right] = \frac{3}{2} \sin \frac{\pi}{2} + \frac{3\pi}{4} = \frac{3}{2} + \frac{3\pi}{4} = \frac{6 + 3\pi}{4}$$



$$D = \{(x,y) \in \mathbb{R}^2 : x \geq 0, y \leq x, x^2 + y^2 \geq 2x, x^2 + y^2 \leq 4x\}$$



$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$= \cos^2 \theta - (1 - \cos^2 \theta)$$

$$= 2 \cos^2 \theta - 1$$

Exercício 7.8 Calcule o volume dos sólidos limitados

- pelos planos definidos por $x = 0$, $x = 1$, $y = 0$, $y = 1$ e $z = 0$ e pela superfície definida por $z = x^2 + y^2$;
- pela superfície definida por $z = \sin y$ e pelos planos definidos por $x = 1$, $x = 0$, $y = 0$, $y = \frac{\pi}{2}$ e $z = 0$;
- pelo parabolóide definido por $z = 4 - x^2 - y^2$ e pelo plano definido por $z = 0$.

$$z = 4 - x^2 - y^2, \quad z = 0$$

$$4 - x^2 - y^2 = 0 \Leftrightarrow x^2 + y^2 = 4$$

$$\int (4 - x^2 - y^2) d(x, y)$$

$$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (4 - x^2 - y^2) dy dx$$

$$r = r \cos \theta \quad 4 - x^2 - y^2 = 0 \Rightarrow 4 - r^2 = 0 \Rightarrow r = 2$$

$$j = r \sin \theta$$

$$\int_0^{2\pi} \int_0^2 (4 - r^2) r dr d\theta = \int_0^{2\pi} \left[\frac{4r^2}{2} - \frac{r^4}{4} \right]_0^2 d\theta = 2\pi (8 - 4) = 8\pi$$

