Matemática Discreta

teoria de números — exercícios

1. Calcule, caso exista, $a^{-1} \mod n$, com

   (a) $a = 14, n = 57$.
   (b) $a = 22, n = 55$.
   (c) $a = 69, n = 96$.
   (d) $a = 32, n = 65$.
   (e) $a = 28, n = 54$.
   (f) $a = 29, n = 84$.
   (g) $a = 53, n = 92$.
   (h) $a = 36, n = 88$.
   (i) $a = 26, n = 81$.
   (j) $a = 31, n = 60$.
   (k) $a = 40, n = 69$.
   (l) $a = 49, n = 67$.
   (m) $a = 83, n = 86$.
   (n) $a = 38, n = 65$.
   (o) $a = 78, n = 82$.
   (p) $a = 53, n = 100$.
   (q) $a = 17, n = 100$.
   (r) $a = 9, n = 82$.
   (s) $a = 45, n = 57$.

2. Resolva as seguintes congruências:

   (a) $81x \equiv 37 \mod 95$.
   (b) $45x \equiv 32 \mod 65$.
   (c) $6x \equiv 46 \mod 52$.
   (d) $26x \equiv 78 \mod 94$.
   (e) $11x \equiv 1 \mod 70$.
   (f) $15x \equiv 37 \mod 82$.
   (g) $46x \equiv 52 \mod 59$.

(h) $24x \equiv 60 \mod 97$.

(i) $7x \equiv 73 \mod 87$.

(j) $40x \equiv 32 \mod 59$.

(k) $60x \equiv 35 \mod 89$.

(l) $2x \equiv 31 \mod 64$.

(m) $21x \equiv 3 \mod 53$.

(n) $22x \equiv 78 \mod 100$.

(o) $76x \equiv 52 \mod 92$.

(p) $25x \equiv 10 \mod 54$.

(q) $27x \equiv 28 \mod 74$.

(r) $28x \equiv 1 \mod 87$.

(s) $77x \equiv 55 \mod 89$.

3. Encontre a solução simultânea de

(a) $x \equiv 16 \mod 19, x \equiv 8 \mod 23, x \equiv 2 \mod 29$.

(b) $x \equiv 10 \mod 11, x \equiv 15 \mod 17, x \equiv 22 \mod 23$.

(c) $x \equiv 9 \mod 17, x \equiv 20 \mod 23, x \equiv 2 \mod 29$.

(d) $x \equiv 11 \mod 11, x \equiv 4 \mod 17, x \equiv 14 \mod 23$.

(e) $x \equiv 13 \mod 19, x \equiv 12 \mod 23, x \equiv 5 \mod 29$.

(f) $x \equiv 3 \mod 5, x \equiv 2 \mod 11, x \equiv 12 \mod 17$.

(g) $x \equiv 3 \mod 11, x \equiv 11 \mod 17, x \equiv 20 \mod 23$.

(h) $x \equiv 16 \mod 19, x \equiv 17 \mod 23, x \equiv 28 \mod 29$.

(i) $x \equiv 16 \mod 19, x \equiv 21 \mod 23, x \equiv 27 \mod 29$.

(j) $x \equiv 1 \mod 5, x \equiv 5 \mod 11, x \equiv 8 \mod 17$.

(k) $x \equiv 8 \mod 11, x \equiv 5 \mod 17, x \equiv 13 \mod 23$.

(l) $x \equiv 4 \mod 11, x \equiv 15 \mod 17, x \equiv 11 \mod 23$.

(m) $x \equiv 3 \mod 7, x \equiv 10 \mod 11, x \equiv 3 \mod 17$.

(n) $x \equiv 13 \mod 17, x \equiv 8 \mod 23, x \equiv 25 \mod 29$.

(o) $x \equiv 2 \mod 5, x \equiv 9 \mod 11, x \equiv 15 \mod 17$.

(p) $x \equiv 11 \mod 17, x \equiv 18 \mod 23, x \equiv 6 \mod 29$.

(q) $x \equiv 15 \mod 17, x \equiv 3 \mod 23, x \equiv 20 \mod 29$.

(r) $x \equiv 5 \mod 17, x \equiv 12 \mod 23, x \equiv 20 \mod 29$.

(s) $x \equiv 2 \mod 7, x \equiv 4 \mod 11, x \equiv 7 \mod 17$.

4. Encontre a solução simultânea de

(a) $16x \equiv 1 \mod 17, 6x \equiv 6 \mod 19, 3x \equiv 4 \mod 23$.

(b) $1x \equiv 2 \mod 3, 3x \equiv 4 \mod 5, 1x \equiv 1 \mod 7$.

(c) $8x \equiv 7 \mod 17, 6x \equiv 7 \mod 19, 3x \equiv 7 \mod 23$.

(d) $4x \equiv 3 \mod 5, 4x \equiv 4 \mod 7, 11x \equiv 7 \mod 11$.

(e) $1x \equiv 5 \mod 7, 2x \equiv 11 \mod 11, 6x \equiv 5 \mod 13$.

(f) $7x \equiv 5 \mod 7, 3x \equiv 5 \mod 11, 3x \equiv 7 \mod 13$.

(g) $2x \equiv 2 \mod 2, 1x \equiv 2 \mod 5, 5x \equiv 3 \mod 7$.

(h) $1x \equiv 3 \mod 3, 2x \equiv 2 \mod 5, 5x \equiv 4 \mod 7$.

(i) $13x \equiv 15 \mod 23, 11x \equiv 25 \mod 29, 2x \equiv 23 \mod 31$.

(j) $9x \equiv 7 \mod 11, 5x \equiv 10 \mod 13, 3x \equiv 10 \mod 17$.

(k) $1x \equiv 5 \mod 23, 23x \equiv 6 \mod 29, 6x \equiv 7 \mod 31$.

(l) $6x \equiv 6 \mod 7, 3x \equiv 7 \mod 11, 3x \equiv 12 \mod 13$.

(m) $4x \equiv 1 \mod 11, 5x \equiv 3 \mod 13, 16x \equiv 3 \mod 17$.

(n) $4x \equiv 2 \mod 5, 3x \equiv 2 \mod 7, 7x \equiv 10 \mod 11$.

(o) $2x \equiv 1 \mod 3, 5x \equiv 4 \mod 5, 1x \equiv 7 \mod 7$.

(p) $3x \equiv 1 \mod 3, 2x \equiv 2 \mod 5, 4x \equiv 1 \mod 7$.

(q) $12x \equiv 9 \mod 13, 17x \equiv 14 \mod 17, 19x \equiv 14 \mod 19$.

(r) $18x \equiv 5 \mod 23, 4x \equiv 11 \mod 29, 14x \equiv 3 \mod 31$.

(s) $2x \equiv 5 \mod 5, 1x \equiv 2 \mod 7, 6x \equiv 11 \mod 11$

5. Calcule $a^k \mod p$, sabendo que p é primo, em que

(a) $a = 7, k = 159, p = 43$.

(b) $a = 2, k = 96, p = 41$.

(c) $a = 6, k = 51, p = 17$.

(d) $a = 4, k = 82, p = 31$.

(e) $a = 4, k = 78, p = 23$.

(f) $a = 3, k = 67, p = 17$.

(g) $a = 9, k = 70, p = 19$.

(h) $a = 9, k = 68, p = 19$.

(i) $a = 3, k = 61, p = 17$.

(j) $a = 7, k = 110, p = 31$.

(k) $a = 16, k = 111, p = 43$.

(l) $a = 11, k = 59, p = 29$.

(m) $a = 2, k = 52, p = 19$.

(n) $a = 2, k = 150, p = 43$.

(o) $a = 9, k = 108, p = 43$.

(p) $a = 3, k = 50, p = 23$.

(q) $a = 13, k = 80, p = 29$.

(r) $a = 7, k = 66, p = 17$.

(s) $a = 6, k = 26, p = 13$.

6. Calcule $\varphi(n)$ dada a factorização em produtos de potências de primos de n:

- $n = 2^4 \cdot 79$.
- $n = 5 \cdot 653$.
- $n = 3^3 \cdot 11^2$.
- $n = 2^2 \cdot 11 \cdot 67$.
- $n = 2^3 \cdot 3^3 \cdot 5$.
- $n = 3 \cdot 17 \cdot 67$.
- $n = 2 \cdot 5 \cdot 157$.
- $n = 2^3 \cdot 3^2 \cdot 5^2$.
- $n = 2 \cdot 3 \cdot 5 \cdot 37$.
- $n = 2 \cdot 17 \cdot 79$.
- $n = 2^2 \cdot 3 \cdot 269$.
- $n = 2 \cdot 5 \cdot 7 \cdot 41$.
- $n = 7 \cdot 227$.
- $n = 2^3 \cdot 17^2$.
- $n = 2 \cdot 3 \cdot 593$.
- $n = 2^2 \cdot 547$.
- $n = 29 \cdot 43$.
- $n = 3 \cdot 509$.
- $n = 2 \cdot 3^2 \cdot 107$.
- $n = 3^2 \cdot 7 \cdot 17 \cdot 23$.
- $n = 3^2 \cdot 13 \cdot 193$.
- $n = 2^2 \cdot 7 \cdot 1381$.
- $n = 15787$.
- $n = 5 \cdot 67 \cdot 97$.
- $n = 2^3 \cdot 2671$.
- $n = 2 \cdot 10069$.
- $n = 2 \cdot 23 \cdot 563$.
- $n = 61 \cdot 293$.

- $n = 2^5 \cdot 1181$.
- $n = 27481$.
- $n = 2^3 \cdot 3 \cdot 5 \cdot 149$.
- $n = 5 \cdot 73 \cdot 109$.
- $n = 2 \cdot 11 \cdot 13 \cdot 131$.
- $n = 2 \cdot 3 \cdot 2399$.
- $n = 2 \cdot 6067$.
- $n = 3^3 \cdot 7 \cdot 151$.
- $n = 3 \cdot 7 \cdot 1237$.
- $n = 2 \cdot 7 \cdot 1693$.

7. Calcule o resto da divisão inteira de a^k por n, onde

   (a) $a = 3, k = 129, n = 40$.
   (b) $a = 7, k = 100, n = 36$.
   (c) $a = 7, k = 84, n = 30$.
   (d) $a = 3, k = 72, n = 28$.
   (e) $a = 5, k = 88, n = 24$.
   (f) $a = 6, k = 130, n = 35$.
   (g) $a = 7, k = 84, n = 30$.
   (h) $a = 5, k = 77, n = 32$.
   (i) $a = 4, k = 97, n = 35$.
   (j) $a = 2, k = 95, n = 35$.
   (k) $a = 4, k = 117, n = 39$.
   (l) $a = 5, k = 151, n = 38$.
   (m) $a = 5, k = 95, n = 28$.
   (n) $a = 2, k = 87, n = 33$.
   (o) $a = 4, k = 84, n = 27$.
   (p) $a = 3, k = 107, n = 28$.
   (q) $a = 8, k = 127, n = 33$.
   (r) $a = 5, k = 131, n = 39$.
   (s) $a = 2, k = 31, n = 15$.

8. Calcule $k! \mod p$, sabendo que p é primo, onde

   (a) $k = 15, p = 17$.
   (b) $k = 9, p = 11$.

(c) $k = 26, p = 31$.

(d) $k = 23, p = 29$.

(e) $k = 17, p = 19$.

(f) $k = 34, p = 37$.

(g) $k = 20, p = 23$.

(h) $k = 36, p = 39$.

(i) $k = 18, p = 23$.

(j) $k = 16, p = 19$.

(k) $k = 19, p = 23$.

(l) $k = 8, p = 11$.

(m) $k = 28, p = 31$.

9. Calcule o resto da divisão de $k!$ quando dividido por n, onde

(a) $k = 10, n = 77$.

(b) $k = 12, n = 65$.

(c) $k = 9, n = 35$.

(d) $k = 16, n = 221$.

(e) $k = 14, n = 143$.

(f) $k = 15, n = 221$.